LIMIT COMPARISON TEST

$L \in \mathbb{R}$ IE. $L \neq \infty$ OR $-\infty$

IF $0 < a_n, b_n$ AND $\lim_{n\to\infty} \frac{a_n}{b_n} = L \neq 0$

THEN $\sum_{n=1}^{\infty} a_n$ CONV IF AND ONLY IF $\sum_{n=1}^{\infty} b_n$ CONV

eg. RECONSIDER $\sum_{n=1}^{\infty} \frac{n-1}{3n^2+1}$

$$\frac{n-1}{3n^2+1} < \frac{n}{3n^2} = \frac{1}{3n}$$

$\frac{n-1}{3n^2+1}, \frac{1}{3n} > 0$ IF $n > 1$

↑ TERMS OF SEQ $\{0, \frac{1}{13}, \frac{2}{28}, \ldots\}$

$$\lim_{n\to\infty} \frac{\frac{n-1}{3n^2+1}}{\frac{1}{3n}} = \lim_{n\to\infty} \frac{(n-1)3n}{3n^2+1} = \lim_{n\to\infty} \frac{3n^2-3n}{3n^2+1} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$\frac{\infty}{\infty}$ L'H OR

$$= \lim_{n\to\infty} \frac{3-\frac{3}{n}}{3+\frac{1}{n^2}} = \frac{3-0}{3+0} = 1 \neq 0$$

AS $n\to\infty$, TERMS IN $\{a_n\}$ ARE GETTING CLOSER + CLOSER TO 1 * TERMS IN $\{b_n\}$

$\sum_{n=1}^{\infty} \frac{1}{3n} = \frac{1}{3} \sum_{n=1}^{\infty} \frac{1}{n}$ DIV (p-SERIES)

$p = 1 \leq 1$

SO $\sum_{n=1}^{\infty} \frac{n-1}{3n^2+1}$ DIV (LIMIT COMPARISON TEST)

P-SERIES
GEOMETRIC
DIVERGENCE
LIMIT COMPARISON
COMPARISON
INTEGRAL

DOES $\sum_{n=1}^{\infty} \frac{\sqrt{2n+5}}{5n^2-3}$ CONV/DIV ?

$$\underbrace{\frac{\sqrt{2n+5}}{5n^2-3}}_{a_n} \quad \cancel{\frac{\sqrt{2n}}{5n^2}} \quad \underbrace{\boxed{\frac{\sqrt{n}}{n^2}}}_{b_n} = \frac{n^{\frac{1}{2}}}{n^2} = \frac{1}{n^{\frac{3}{2}}}$$

$$\frac{\sqrt{2n+5}}{5n^2-3}, \ \frac{1}{n^{\frac{3}{2}}} > 0$$

$$\boxed{\lim_{n\to\infty} \frac{\sqrt{2n+5}}{5n^2-3} \cdot \frac{n^2}{\sqrt{n}}} = \lim_{n\to\infty} \sqrt{\frac{2n+5}{n}} \cdot \frac{n^2}{5n^2-3} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \boxed{\lim_{n\to\infty} \sqrt{\frac{2+\frac{5}{n}}{1}} \cdot \frac{1}{5-\frac{3}{n^2}}}$$

$$\frac{\frac{1}{n}}{\frac{1}{n}}$$

$$= \sqrt{\frac{2+0}{1}} \cdot \frac{1}{5-0}$$

$$= \boxed{\frac{\sqrt{2}}{5}} \ \boxed{\neq 0}$$

$\boxed{\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}} \text{ CONV}}$ $\boxed{\text{(p-SERIES TEST)}}$

$\boxed{p = \frac{3}{2} > 1}$

SO, $\boxed{\sum_{n=1}^{\infty} \frac{\sqrt{2n+5}}{5n^2-3} \text{ CONV}}$ $\boxed{\text{(LIMIT COMP TEST)}}$

DOES $\sum_{n=1}^{\infty} \frac{1}{n2^n}$ CONV/DIV?

COMPARE $\frac{1}{n2^n}$ TO $\frac{1}{2^n}$

$\frac{1}{n2^n}, \frac{1}{2^n} > 0$

$\lim_{n\to\infty} \frac{1}{n2^n} \cdot \frac{2^n}{1} = \lim_{n\to\infty} \frac{1}{n} = 0$ ← CAN'T USE LIMIT COMP THIS WAY

COMPARE $\frac{1}{n2^n}$ TO $\frac{1}{n}$

$\frac{1}{n2^n}, \frac{1}{n} > 0$

$\lim_{n\to\infty} \frac{1}{n2^n} \cdot \frac{n}{1} = \lim_{n\to\infty} \frac{1}{2^n} = 0$

$0 < \frac{1}{n2^n} < \frac{1}{2^n}$

$\sum_{n=1}^{\infty} \frac{1}{2^n}$ CONV (GEOMETRIC TEST) SO $\sum_{n=1}^{\infty} \frac{1}{n2^n}$ CONV

$|r| = \frac{1}{2} < 1$

(COMP TEST)

11.5 ALTERNATING SERIES TEST

IF $\{a_n\}$ IS POSITIVE, DECREASING AND CONV TO 0,

THEN $\sum_{n=1}^{\infty}(-1)^{n-1}a_n$ CONV